D2 or M2?

A Note on Membrane Scattering

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ABSTRACT

Motivated by a physical interpretation of its correlation functions as membrane scattering amplitudes, we re-address whether the Lorentzian BLG theory can be quantized such that it preserves $SO(8)$ superconformal symmetry. We find that this appears to be possible. While the model correctly reproduces protected quantities such as chiral primary amplitudes and the four derivative effective action, we conclude that, as understood at present, it gives a relatively unpractical parametrization of the IR dynamics of M2-branes.

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The groundbreaking work of Bagger, Lambert [1][2], and Gustavsson [3] has helped uncover valuable new insight into the structure of the the worldvolume theory of coincident M2-branes. This rapid development culminated in the recent ABJM formulation of the M2-brane worlvolume theory in terms of an $\mathcal{N}=6$ Chern-Simons theory [4].

An earlier proposal for an $SO(8)$ invariant formulation was made in [5][6][7], based on a Lorentzian three-algebra. This model has the required classical symmetries, but has several unresolved problems. In particular, the classical theory has ghosts, $X_\pm$. This feature makes it unclear whether the theory can be quantized in a way that simultaneously preserves unitarity and $SO(8)$ superconformal symmetry.[2] Moreover, the ghost-free formulation seems directly equivalent to the non-conformal D2 theory [9][10][11][12]. As understood at present, Lorentzian model indeed appears to provide an incomplete description of low energy M2-brane physics [13]. Nonetheless, even with this current assessment, it is worthwhile to examine the reach and limitations of the model, and possibly identify a class of physical questions where it can be useful.

This short note re-addresses the question whether the Lorentzian model can be quantized such that its amplitudes are non-trivial, $SO(8)$ invariant, and exhibit superconformal symmetry. Relative to earlier discussions, we introduce two new ingredients:

(i) To decouple negative norm states, the X_+ ghost field needs to satisfy its equations of motion, $\partial^2 X_+ = 0$. To get non-trivial correlation functions, we impose this equation of motion everywhere except at the locations z_i of the local operators $\mathcal{O}_i(z_i)$. Hence X_+ is allowed to develop a pole at the z_i

$$X_+(y) = \sum_i \frac{\mathsf{q}_i}{|y - z_i|} \,, \tag{1}$$

with q_i some (initially) constant $SO(8)$ vectors. Simultaneously, as indicated in fig 1, we will choose a world volume metric such that the region around the points z_i takes the form $S^2 \times \mathbb{R}$. The Lorentzian 3-algebra theory then describes some generalized version of 2+1 SYM theory, with certain $SO(8)$-twisted boundary conditions and with a position dependent effective coupling g_{YM} proportional to the q_i.

Via the interpretation of X_+ proposed in [14], as a radial center of mass coordinate of the membrane stack, this prescription makes the correlation functions look like scattering amplitudes of asymptotic multi-membrane states. For fixed values of the q_i parameters, $SO(8)$ superconformal symmetry is broken, in the same way that any given scattering amplitude breaks the space-time symmetries. To get amplitudes that can be interpreted as $SO(8)$ SCFT correlation functions, one would need to decouple the q_i.

(ii) By adding an appropriate ghost action, we show that all negative norm states decouple, provided that the q_i parameters are treated as dynamical variables. We will arrange the ghost action such that the q_i in fact must attain their saddle point values.

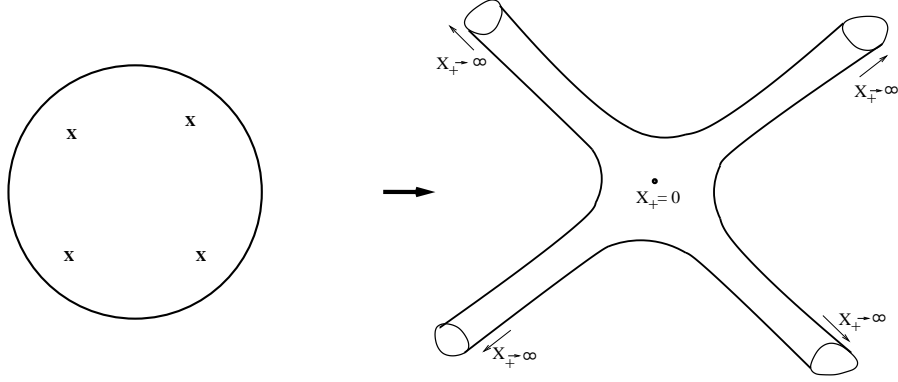[2]The proposal in the first version of [11] was incomplete, and has been retracted.

Figure 1: X_+ is allowed to develop a pole at the location of local operators. Via the proposed identification X_+ as a center of mass coordinate [14], correlation functions thus have a suggestive interpretation as scattering amplitudes of asymptotic multi-membrane states.

Correlation functions thus take the form of a discrete sum

$$\mathcal{A} = \sum_{X_+^{\mathrm{cl}}} \left\langle \prod_i \mathcal{O}_i(z_i) \right\rangle_{X_+^{\mathrm{cl}}} \tag{2}$$

where $\left\langle \ldots \right\rangle_{X_+^{\mathrm{cl}}}$ denotes the amplitude computed at $X_+ = X_+^{\mathrm{cl}}$, with X_+^{cl} a saddle point value, at which the correlation function is extremized

$$\frac{\delta}{\delta X_+} \left\langle \prod_i \mathcal{O}_i(z_i) \right\rangle \Big|_{X_+ = X_+^{\mathrm{cl}}} = 0 \,. \tag{3}$$

The conformal fixed point value of X_+ is evidently one of the saddle points, but a given correlation function may have other extrema.

The proposed formula (2)-(3) has some positive features: it does not involve any integral, and formally preserves $SO(8)$ and conformal invariance. It nonetheless appears to have somewhat limited practical use. The fixed point is expected to occur for large X_+, where the lagrangian becomes strongly coupled. Suitably protected quantities, however, such as correlation functions of chiral primary operators and special higher derivative terms in the effective action, are still accessible to computation and are correctly reproduced [15].

At a technical level, one could ask whether the prescription of extremizing w.r.t. the parameters q_i makes sense. Should we treat them like vevs, and keep them fixed? Or, since they are determined by local boundary conditions, are we allowed to treat them as dynamical variables? We return to this question after we have summarized the formulation that leads to the mentioned results.

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Membrane Scattering, Take I

The Lorentzian BLG action is a sum of kinetic terms (we choose the gauge $D^\mu B_\mu = 0$)

$$\mathcal{L}_0 = \mathrm{Tr}\left(-\frac{1}{2}(D_\mu X^I)^2 + \frac{i}{2}\bar{\Psi}\slashed{D}\Psi + \frac{1}{2}\epsilon^{\mu\nu\lambda}B_\mu F_{\nu\lambda}\right)$$

$$\mathcal{L}_\pm = \partial^\mu X_-\partial_\mu X_+ - \frac{i}{2}\left(\bar{\Psi}_-\slashed{\partial}\Psi_+ - h.c.\right),$$

(4)

and an interaction term

$$\mathcal{L}_{int} = -\frac{1}{2}(X_+^I)^2(B_\mu)^2 - \frac{1}{12}\mathrm{Tr}\left(X_+^I[X^J,X^K]+X_+^J[X^K,X^I]+X_+^K[X^I,X^J]\right)^2$$

$$+ \frac{i}{2}\mathrm{Tr}\left(\bar{\Psi}\Gamma_{IJ}X_+^I[X^J,\Psi]\right) + \frac{i}{4}\mathrm{Tr}\left(\bar{\Psi}\Gamma_{IJ}[X^I,X^J]\Psi_+\right) + h.c.$$

(5)

The kinetic term for the ghost fields $X_\pm$ and $\Psi_\pm$ is non-positive definite.

The above lagrangian is $SO(8)$ invariant. For any non-zero value of X_+, however, $SO(8)$ invariance is spontaneously broken: the eight scalar fields X^I split up in seven transverse components and one longitudinal component in the direction of X_+. For constant X_+, the Lorentzian theory reduces to the D2 world volume theory, and the transverse components become the seven scalars of 2+1 SYM, and the longitudinal component is the magnetic dual scalar to the non-abelian gauge field A, [11].

The X_+ field satisfies the free field equations

$$\partial^2 X_+ = 0.$$

(6)

The space of such solutions depends on boundary conditions. It is therefore useful to contemplate the physical interpretation of X_+.

From the world volume perspective, X_+ governs the effective SYM coupling constant via $g_{YM}^2 = (X_+^I)^2$. To get a new and non-trivially interacting theory, we thus need to choose boundary conditions such that X_+ is non-vanishing and non-constant. From the target space perspective, X_+ appears to behave as a center of mass position of the stack of membranes [14], as defined by averaging the locations of all branes within the stack. This average can be defined locally on the collective world volume, and thus the center of mass location may vary with the world volume coordinates.

With this motiation, we will allow the $X_+(z)$ field to develop a pole at the location z_i of the local operators as given in eqn (1). The coefficient q_i of the pole at z_i can be interpreted as the center of mass momentum of the multi-membrane state created by the operator at the location z_i. Simultaneously, we will define the local operators at a point z via 'radial quantization': we use the log of the radial distance to the point z as 'time',

and construct the Hilbert space by expanding the fields in spherical harmonics on the S^2 surrounding z. Concretely, this means that, for computing correlation functions

$$\mathcal{A} = \left\langle \prod_i V_{\mathcal{O}_i}(z_i, \mathsf{q}_i) \right\rangle, \tag{7}$$

we will not choose a standard flat metric, but take it of the form

$$ds^2 = \sigma^2(y) dy^\mu dy_\mu, \qquad \sigma(y) = \sum_i \frac{R}{|y - z_i|}. \tag{8}$$

In this metric, the worldvolume develops a tube like region of the form $S^2 \times \mathbb{R}$ near every operator insertion z_i. Here R denotes the radius of the S^2.[3]

To define correlation functions, we need to introduce suitable class of local operators. As a first reasonable guess, we write the vertex operators that create the asymptotic states in the factorized form

$$V_{\mathcal{O}}(z, \mathsf{q}) = \mathcal{O}(z) \, e^{i\mathsf{q} \cdot X_-(z)} \tag{9}$$

where $\mathcal{O}(z)$ is a local operator defined out of the transverse fields (i.e. the fields that for fixed and constant X_+ constitute 2+1 SYM theory). Examples of such local operators $\mathcal{O}(z)$ are the chiral primary operators described in [11].

The X_+-field develops a pole near the vertex operator (9). Since we are using the metric (8), this does not mean that the effective SYM coupling blows up near z. To see this, we can look at the vertex operator (9) as a state on $S^2 \times \mathbb{R}$. Applying radial quantization to $X_\pm$ yields a set of creation and annihilation modes $a^\dagger_{\pm,\ell m}$, $a_{\pm,\ell m}$. The q_i modes in (1) correspond to constant zero modes $a_{+,0}$. The state created by the vertex operator (9) factorizes as

$$|V_{\mathcal{O}}\rangle = |\mathcal{O}\rangle |0, \mathsf{q}\rangle \tag{10}$$

where $|0, \mathsf{q}\rangle$ is annihilated by all annihilation modes with $\ell > 0$, while for the zero modes

$$a_{-,0} |0; \mathsf{q}\rangle = 0, \qquad a_{+,0} |0; \mathsf{q}\rangle = \mathsf{q} |0; \mathsf{q}\rangle. \tag{11}$$

In this state, the theory locally reduces to SYM on $S^2 \times \mathbb{R}$ with coupling $g^2_{\text{YM}} = \mathsf{q}^2/R^2$, or in dimensionless units, $g^2_{\text{YM,eff}} = \mathsf{q}^2/R$. Constructing the quantum operators $V_{\mathcal{O}}(z, \mathsf{q})$ thus still requires control over the interacting 2+1 SYM theory. For BPS operators, this problem appears to be tractable.

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[3]We assume that all fields are conformally coupled, so that the action is Weyl invariant. The original and Weyl rescaled X_+ field are related via $X_+ = \sigma \widetilde{X}_+$. Here X_+ satisfies the free eqn of motion (6) and may develop poles as in (1), while $\widetilde{X}_+$ remains finite at z_i.

All this would seem to yield a satisfactory prescription for computing amplitudes, with the expected symmetries to support the interpretation as scattering of multi-membrane states in a flat 11-d space-time. The individual vertex operators (9) are $SO(8)$ covariant: the $SO(8)$ symmetry is broken to $SO(7)$ by the direction of the center of mass momentum q. However, there is the problem of unitarity. The lagrangian (4) has a non-positive definite kinetic term, and while the specific vertex operators (9) produce positive norm states, the above prescription does not provide a mechanism for proving that negative norm states can be consistently decoupled from physical processes.

Adding Ghosts

To eliminate negative norm states, it was proposed in [11] that it is sufficient to introduce a free, non-interacting ghost sector $(c_\pm, \chi_\pm)$. This procedure indeed gives rise to a unitary model. Ghost fields, however, should also serve to provide a proper integration measure on the classical configuration space, and the minimal set-up of [11] appears to be insufficient in this respect. Here we will introduce a ghost sector that not only eliminates negative norm states, but also helps define the X_+ integral. In fact, it reduces it to a sum over saddle points as in (2).

As a quick guide to the following discussion, here's a toy example of an integral that reduces to a sum over saddle points

$$\mathcal{Z} = \int dx\, dx^*\, d\theta\, d\theta^*\, e^{iS(y) + ix^* S'(y)}, \qquad y = x + \frac{\theta \theta^*}{x^*}, \tag{12}$$

where θ and θ^* are anti-commuting. Performing the θ, θ^* and x^* integration gives[4]

$$\mathcal{Z} = \int dx \left(\tfrac{i}{2} |S'(x)| + S''(x) \delta(S'(x)) \right) e^{iS(x)} = 2 \sum_{x_{\mathrm{cl}}} e^{iS(x_{\mathrm{cl}})} \,.$$

The path integral over our ghost sector will have a form very similar to (12).

With this motivation, we extend the ghost sector by 'complexifying' all ghost fields $X_\pm$, $\Psi_\pm$, $c_\pm$ and $\chi_\pm$. The complex conjugate fields will be denoted by $X_\pm^*$, etc, but should be viewed as independent fields from $X_\pm$, etc. The free ghost action reads

$$\mathcal{L}_\pm = \partial^\mu X_-^* \partial_\mu X_+ - i \bar{\Psi}_-^* \slashed{\partial} \Psi_+ \ + \ h.c.$$

$$\mathcal{L}_{\mathrm{gh}} = \partial^\mu c_- \partial_\mu c_+ - i \bar{\chi}_- \slashed{\partial} \chi_+ \ + \ h.c. \tag{13}$$

and has $SO(8)$ supersymmetry. The $c_\pm$ and $\chi_\pm$ fields have opposite statistics to their counter parts $X_\pm$ and $\Psi_\pm$, and are designed to cancel their quantum fluctuations.

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[4]Here we used $\int dx^* e^{ix^* p}/x^* = i\pi \operatorname{sign}(p)$, and dropped a factor of $2\pi i$. The sum is only over minima of $S(x)$.

In principle, we could try to derive this ghost sector by applying the rules of BRST quantization to some proper gauged version of the Lorentzian 3-algebra lagrangian. We will not attempt to do so here. Instead, we just introduce all the ghost fields by hand, and postulate that the physics needs to be invariant under the nilpotent BRST transformations

$$\delta_Q X_- = \varepsilon c_-, \qquad \delta_Q X_+ = \varepsilon c_+^*, \qquad \delta_Q \Psi_- = \varepsilon \chi_-, \qquad \delta_Q \Psi_+ = \varepsilon \chi_+^*,$$

$$\delta_Q c_-^* = \varepsilon X_-^*, \qquad \delta_Q c_+ = \varepsilon X_+^*, \qquad \delta_Q \chi_-^* = \varepsilon \Psi_-^*, \qquad \delta_Q \chi_+ = \varepsilon \Psi_+^*. \tag{14}$$

Defining physical states by the Q cohomology eliminates all negative norm states. To ensure BRST invariance of the full theory, however, it is necessary to add an extra interaction term $\Delta \mathcal{L}_{int}$ to the ghost action. The new term should also preserve supersymmetry. These two conditions are highly restrictive, though still allow for more than one solution. Different solutions differ by Q exact terms, and correspond to different choices for the 'gauge fixing fermion'.

The toy example (12) suggests that we write the new interaction lagrangian as

$$\mathcal{L}_{int}(Y_+, \Upsilon_+) + \left\{ Q, c_+^I P_I + \chi_+^a S_a \right\}, \tag{15}$$

$$P_I = \frac{\partial \mathcal{L}}{\partial X_+^I}, \qquad S_a = \frac{\partial \mathcal{L}}{\partial \Psi_+^a}.$$

Here $\mathcal{L}_{int}$ is the same expression as the old interaction lagrangian, with X_+ and Ψ_+ replaced by Q invariant modifications Y_+ and Υ_+.

The new action continues to be invariant under $SO(8)$ supersymmetry, provided that the new variables form a proper supermultiplet

$$\delta_{\text{susy}} Y_+^I = i\bar{\epsilon} \Gamma^I \Upsilon_\pm, \qquad \delta_{\text{susy}} \Upsilon_+ = \slashed{\partial} Y_+^I \Gamma^I \epsilon. \tag{16}$$

These conditions determine the modified fields Y_+ and Υ_+, modulo Q exact terms. A minimal solution is to take Y_+ of the form

$$Y_+^I = X_+^I + c_+^{*I} c_+^K v^K, \qquad v^I = \frac{X_+^{*I}}{(X_+^*)^2}, \tag{17}$$

and solve for Υ_+ using (16). We will not need the explicit expression for Υ_+.

The action (15) with Y_+ as in (17) looks similar to the exponential in (12). The second, Q exact term in (15) has the standard form of a gauge fixing term, associated with the gauge fixing conditions $P_I = 0$ and $S_a = 0$. These gauge fixing conditions can be recognized as the saddle point equations for X_+ and Ψ_+, and it should therefore be no surprise that in the end, the integral over the ghost fields reduces to the classical sum (2).

Boundary Conditions at Local Operators

With the extended ghost sector and BRST symmetry in hand, we can now revisit the definition of local operators, and make sure that all negative norm states decouple. Applying radial quantization to the ghost sector $(X_\pm, c_\pm)$ gives a set of creation and annihilation modes $a^\dagger_{\pm,\ell m}$, $a_{\pm,\ell m}$, $c^\dagger_{\pm,\ell m}$ and $c_{\pm,\ell m}$, and similar for the complex conjugate fields. Let us pick some $SO(8)$ direction Ω, and for now just focus on the modes in this direction.

The BRST charge, that generates the nilpotent symmetry (14), can be expanded as

$$Q = c_{-,0}a^{*\dagger}_{+,0} - c^\dagger_{-,0}a^*_{+,0} + c^*_{+,0}a^{*\dagger}_{-,0} - c^{*\dagger}_{+,0}a^*_{-,0} + \ldots$$

where the ellipsis denote oscillators with $\ell \geq 1$. The Q cohomology is trivial: the only obvious physical ghost sector state is the vacuum $|0\rangle$ annihilated by all annihilation modes. Via the state operator map, it represents the identity operator 1.

However, there are other possible physical vacua. We can consider vacuum states annihilated by all annihilation modes with $\ell \geq 1$ and by all zero modes of the minus fields:

$$a_{-,0}|-1\rangle = 0, \qquad a^\dagger_{-,0}|-1\rangle = 0, \qquad c_{-,0}|-1\rangle = 0, \qquad c^\dagger_{-,0}|-1\rangle = 0, \qquad (18)$$

and similar for the complex conjugate zero modes.[5] This state $|-1\rangle$ is Q invariant, but is not annihilated by the annihilation zero modes of the plus fields. The path-integral thus includes the integral over the corresponding modes. We can make this integral explicit via

$$|-1\rangle = \int d\mathsf{q}\, c^\dagger_{-,0}|0,\mathsf{q}\rangle \times h.c. \qquad (19)$$

where $|0,\mathsf{q}\rangle$ is the state with given center of mass momentum q defined in (11).

The state $|-1\rangle$ is annihilated by all lowering operators of the superconformal algebra, but unlike the standard vacuum $|0\rangle$, not by all superconformal generators. Instead, it forms the lowest component of a supermultiplet.

Via the operator state map, $|-1\rangle$ corresponds to the local vertex operator

$$V_{-1}(z) = \int d\mathsf{q}\, c_-(z)e^{i\mathsf{q}\cdot X_-(z)} \times h.c.. \qquad (20)$$

This operator eliminates the functional integration over the value of the minus fields X_-, X^*_-, c_- and c^*_- at the location z. It thereby frees up the integration over the plus modes, that behave as $1/|y - z|$. (We'll write these modes momentarily.) $V_{-1}(z)$ is not invariant under supersymmetry, but instead forms the lowest component of a superfield.

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[5]The $|-1\rangle$ vacuum is obtained from $|0\rangle$ by a shift in the Fermi and Bose sea level. This type of vacuum is familiar from superstring theory, where vertex operators are most naturally inserted in the -1 picture. The ghost insertion reflect the condition that the gauge transformations vanish at z_i. This restriction creates an extra modulus, that needs to be integrated over.

Local operators at the point z take the form

$$V_{\mathcal{O}}(z) = \mathcal{O}(z)V_{-1}^{\Omega}(z), \qquad (21)$$

where $\mathcal{O}(z)$ denotes the lowest superfield component of any local gauge invariant operator made up from the transverse modes. In (21) we have given $V_{-1}^{\Omega}(z)$ the superscript Ω to indicate that it involves a particular choice of direction within $\mathbb{R}^8$. The full superfield version of $V_{\mathcal{O}}(z)$ is given by the product of the superfields of $\mathcal{O}(z)$ and $V_{-1}^{\Omega}(z)$. Note that this means that the boundary conditions on the ghost fields depends on which component of the matter supermultiplet one considers. When defined in this way, the boundary conditions at the operator locations preserve supersymmetry.

Correlation Functions

Correlation functions are obtained inserting the physical operators (21) in the path integral of the Lorentzian BLG model, extended with the new ghost sector

$$\left\langle \prod_i \mathcal{O}_i(z_i) \right\rangle = \mathcal{N} \int \mathcal{D}\,[\text{Fields}]\, e^{-S[\text{Fields}]} \prod_i V_{\mathcal{O}_i}(z_i) \qquad (22)$$

with $\mathcal{N}$ some overall normalization factor, chosen such that $\langle\, 1\, \rangle = 1$. The right-hand side is computed with the metric (8).

The functional integral can be factorized into an integration over the complexified ghost sector times an integral over the transverse modes. Since all the ghost fields are non-interacting, it is straightforward to do their integral. The minus ghost fields are absent from all interactions and observables, except in the V_{-1} operators. The V_{-1} insertions have the effect of freeing up the integration over the singular modes

$$X_+(y) = \sum_i \mathsf{q}_i\, \mu_i(y), \qquad c_+(y) = \sum_i \hat{\mathrm{C}}_i\, \mu_i(y)\,, \qquad \mu_i(y) = \frac{e^{\Omega_i}}{|y - z_i|}\,, \qquad (23)$$

and similar for the complex conjugates fields. Here e^{Ω_i} denotes a unit vector in the $SO(8)$ direction Ω_i, and indicates the $SO(8)$ orientation of the modes associated with q_i and $\hat{\mathrm{C}}_i$.

All regular ghost modes can be integrated out, producing a trivial overall factor of 1. What remains is the finite dimensional integral over the 'moduli', q_i, $\hat{\mathrm{C}}_i$ and their complex conjugates.[6] This integral looks very similar to the toy example (12), and it similarly reduces to the sum (2) over semi-classical saddle points. This result was anticipated, given that the last term in the interaction lagrangian (15) can be viewed as a gauge fixing term, that imposes the saddle point equations as a gauge condition.

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[6]These parameters indeed play a somewhat similar role as the complex structure moduli in the path integral expression for string amplitudes.

We thus arrive at the prescription (2)-(3). Assuming that correlation functions for fixed q_i are finite and reasonably well-behaved, it in principle provides a concrete answer, that preserves all symmetries of the classical Lorentzian 3-algebra lagrangian, in the sense that they are broken only by the local operators insertions. Let us briefly address each of the three symmetries – $SO(8)$ invariance, supersymmetry, and conformal invariance.

The boundary conditions at the local operators imposed by $V_{-1}^{\Omega}(z)$ break $SO(8)$ invariance, since they depend on a choice of direction in $\mathbb{R}^8$. In the interacting theory, the operator $\mathcal{O}(z)$ is sensitive to this choice of orientation. However, we can restore rotation symmetry, by including into the definition of the amplitudes, the integral over all angles Ω_i. In the language of membrane scattering, this integral amounts to performing an s-wave projection on all asymptotic states. While this projection removes one apparent source of $SO(8)$ symmetry breaking, the true origin of $SO(8)$ invariance is that 2+1 SYM theory is expected to flow to an $SO(8)$ invariant IR fixed point SCFT (now conjectured to be described, albeit in a non-manifestly $SO(8)$ invariant way, by the $k = 1$ ABJM theory). The RG flow thus erases, at least locally, the dependence of the local operators on the orientation Ω_i.

Supersymmetry is preserved, provided that one uses the proper superfield version of the vertex operators, given by the product of the superfield of $V_{-1}(z)$ and the superfield of $\mathcal{O}(z)$. The $V_{-1}(z)$ superfield is obtained by replacing c_- and X_- in eqn (20) by their respective superfields. Scale invariance follows from the fact that field that sets the scale, X_+, is dynamically driven to attain its saddle point value.[7]

Discussion

One technical argument one could perhaps raise, is that the moduli q_i and Ω_i should not be treated as dynamical variables, but represent vacuum expectation values that should be held fixed. However, as shown above, the value of the moduli represent properties of a local operator $V_{-1}(z)$, or equivalently, of the corresponding state $|-1\rangle$ defined on S^2. In a Hilbert space defined on a compact spatial slice, one is free to define projection operators. We conclude therefore that one is allowed to integrate over the moduli. Moreover, as we have seen, decoupling of negative norm states in our formulation in fact *requires* that we treat the moduli q_i as dynamical.

Another more fundamental point of criticism is that the Lorentzian model fails to reach its target because the variables used in the lagrangian become strongly coupled for large X_+ and thus provide an unpractical parametrization of the IR physics of M2-branes. In view of the high degree of supersymmetry, however, one could still be optimistic that, either by some clever insight similar to that of matrix string theory in 1+1 dimensions, or by focusing on the right type of quantities, one can still retain control over the theory, or aspects thereof, at large values of X_+.

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[7] A simple analogy is the point particle action, where minimizing the non-scale invariant action $\dot{x}^2/e + m^2 e$ with respect to e gives the scale invariant $m\sqrt{\dot{x}^2}$.

An illustrative example of a non-trivial quantity, that can be explicitly computed and has a well-controlled strong coupling limit, is the four derivative term in the effective action. For 2+1 SYM theory this term has been calculated to all non-perturbative orders in [15]. It involves a single perturbative contribution and an elaborate infinite sum over monopole instanton contributions. Via a Poisson resummation, and translated to the Lorentzian 3-algebra model, the result can be recast in the schematic form

$$\sum_n \frac{\left((\partial X)^2\right)^2}{\left((X_\perp)^2 + (X^8 + nX_+)^2\right)^3} \tag{24}$$

where $X_\perp$ denote the components of the scalar fields X^I perpendicular and X^8 the component parallel to X_+. The above expression has a saddle point value at $X_+ \to \infty$, yielding the $SO(8)$ invariant and conformally invariant answer

$$\frac{\left((\partial_\mu X)^2\right)^2}{\left((X)^2\right)^3}. \tag{25}$$

This result also matches with the expectations derived from the proposed interpretation of the model as describing M2-branes interacting via 11-d supergravity. Although based on old calculations and derived from 2+1 SYM only, this correspondence still represents a non-trivial test of the M2-brane interpretation of the model.

Conclusion

In this note we have addressed some of the technical and practical criticisms of the proposed interpretation of the Lorentzian BLG theory as describing the world volume theory of M2-branes. We believe that the treatment of the model as presented here answers the main technical objections, and also highlights more clearly the added structure relative to the pure D2-brane theory. Nonetheless, the physical content of the Lorentzian model is still very closely linked with that of D2-branes. At a practical level, this rather limits the amount of new physical and quantitative insight that one can extract from it. In its current form, the Lorentzian model clearly does not fully capture the same structure and dynamical aspects of the IR physics of M2-branes, as encoded in the Bagger-Lambert and ABJM lagrangians. Comparing the two perspectives, however, may still be useful as a potential route towards deriving $SO(8)$ invariance of the $k = 1$ ABJM theory.

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